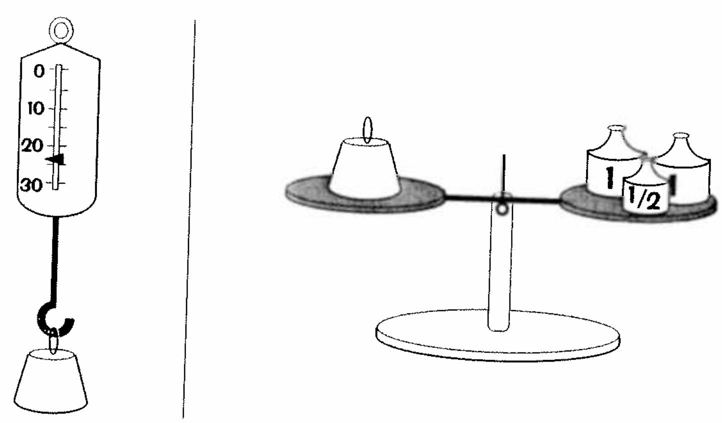
2.2 NEWTON'S LAWS OF MOTION

In 1686, Sir Isaac Newton (1642–1727) published his three laws of motion, which relate the motion of objects (their kinematical behavior or response) to the causes of that motion (the individual forces exerted on the objects). These three laws remain the basis of how scientists view the macroscopic (large-scale) world. We sometimes refer to this view as *Classical Mechanics*.

Mass versus weight. In order to fully understand Newton's laws, we must first make a distinction between the ideas of *mass* and *weight*. Weight is the magnitude of the gravitational force exerted on one object by everything else! Usually this is simply due to the closest celestial object, like the earth or the moon. For most objects a spring scale can be used to measure the object's weight.

Loosely speaking, mass is the "amount of stuff" we have, which does not change when we change locations. It cannot be measured using a spring scale, because then its value would be different at different locations in the universe. Instead, we use an *equal-arm balance* to measure an object's mass.

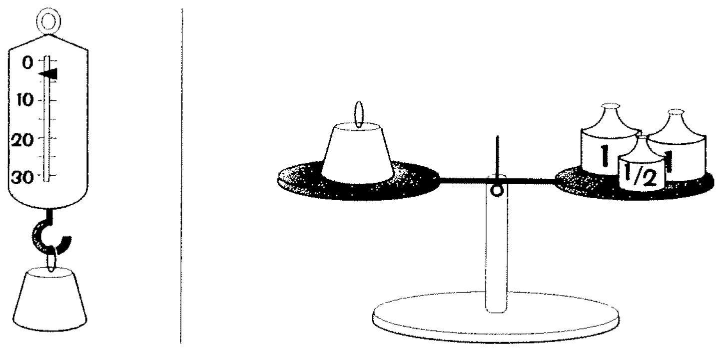
Consider an object that weighs (about) 5½lb on the earth, or (about) 24N. On a spring scale, let's say the spring stretches by 12cm (that is, it stretches 1cm for every 2N of weight it measures). On an equal-arm balance, it requires 2½ standard "units" of mass to balance our object, so we say that the object has a mass of 2½ kilograms (or kg). (That is, the standard unit of mass is the kilogram.)



the weight of an object versus its mass as measured on the surface of the earth

On the surface of the moon, we would find some changes. Using the same spring scale we discover that the moon's gravitational pull is weaker, because the spring only stretches by 2cm (instead of 12cm). Therefore, the weight of our object is only 4N on the moon. However, using the same equal-arm balance, we find that the same number of standard units of mass is

needed to balance our object. This is because the moon pulls less on our standards by the same proportion that it does on our object. This means our measurement of mass is the same everywhere! (As long as there is a gravitational field!!)



the weight of an object versus its mass as measured on the surface of the moon

When the mass is measured using an equal-arm balance, it is sometimes referred to as the *gravitational mass*. In the next section, we will use the *inertial mass* to show how forces affect the motion of objects. For now, we will assume that these two are identical, and we will use them interchangeably. We will use the same symbol m to represent each one.

Newton's three laws of motion. We now present Newton's Laws of Motion, restated in modern English. For each law, we provide an explanation and, if possible, a mathematical description.

NEWTON'S FIRST LAW OF MOTION

> *An object moving at a particular velocity (speed and direction) will remain at that same velocity until an unbalanced force is exerted on it.*

- Remaining at constant velocity means that its motion does not change:
  - (1) An object at rest (stationary) would remain at rest.
  - (2) An object going at a particular speed in a particular direction would simply keep going at that speed in that direction.
- An unbalanced force is needed to change <u>either</u> the speed <u>or</u> the direction of motion.
- When the forces exerted on an object are unbalanced, we say there is a "net force on the object". The net force is the vector sum of all the individual forces exerted on the object.

> *Whenever there is a non-zero net force exerted on an object, the object accelerates in the same direction as the net force. The magnitude of the acceleration is the magnitude of the net force divided by the mass of the object.*

- Mathematically Newton's 2nd law is written:

$$\mathbf{F}_{net} = m\,\mathbf{a}. \qquad \textbf{Newton's 2nd law}$$

where:
$\mathbf{F}_{net}$ = the net force on the object
= the vector sum of all individual forces
m = the mass of the object
$\mathbf{a}$ = the acceleration of the object
= the vector describing the rate at which the velocity of the object changes

- A net force causes an acceleration, which is recognized by a <u>change</u> in velocity.
- Newton's 2nd law tells us that if the mass of an object is constant, then its acceleration is proportional to the net force exerted on it:

$$\mathbf{a} \propto \mathbf{F}_{net} \qquad \text{when } m \text{ is constant.}$$

- The 2nd law also tells us that if the same net force is exerted on different objects, their accelerations are proportional to the inverses of their masses:

$$\mathbf{a} \propto \frac{1}{m} \qquad \text{when } \mathbf{F}_{net} \text{ is constant.}$$

  So, the larger an object's mass, the smaller its acceleration.

- Because the tendency <u>not</u> to change is called *inertia*, the mass used in Newton's 2nd law is a measure of an object's inertia, and is sometimes called its *inertial mass*. The inertial mass is assumed to be equal to the *gravitational mass* (the mass used in the empirical law for the gravitational force).
- Whenever the net force on an object is zero, we say that the object is *in equilibrium*, even if it is moving at constant velocity.

> *Whenever one object exerts a force on a second object, the second object always exerts a force on the first. These two forces are equal in magnitude, are opposite in direction, and have the same nature.*

- Mathematically Newton's 3rd law is written:

  $$\mathbf{F}_{\text{on 1 by 2}} = -\mathbf{F}_{\text{on 2 by 1}}.$$ **Newton's 3rd law**

  where:

  $\mathbf{F}_{\text{on 1 by 2}}$ = the force exerted on object 1 by object 2

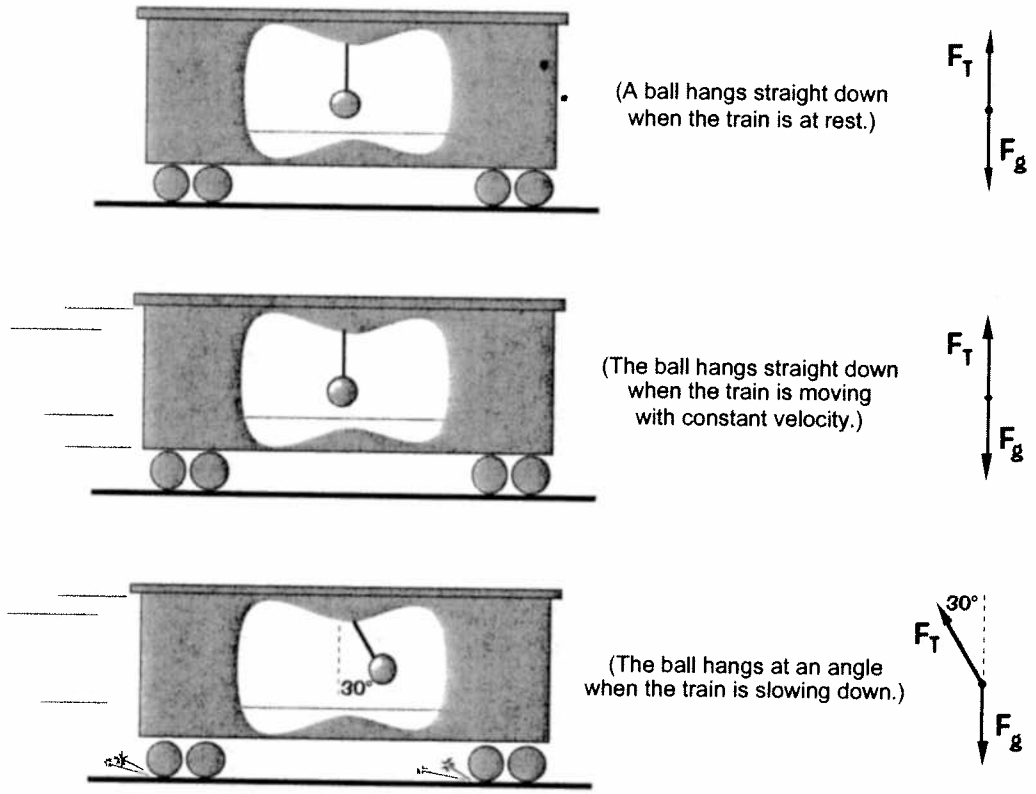
  $\mathbf{F}_{\text{on 2 by 1}}$ = the force exerted on object 2 by object 1

- These two forces are the two sides of a single interaction, so they are always the same kind of force, such as gravitational, frictional, normal, spring, etc.

- These two forces are sometimes referred to as either *an action/reaction pair* or *an action and its reaction*. Since either one of the forces could be the action and the other the reaction, it is incorrect to think that one of them happens first, followed by the other. Both occur at exactly the same time. If one of them changes, then both of them change, instant by instant.

- Although these two forces add up to zero (mathematically), they do not add up to a net force of zero on any one object. Each is exerted on a different object, so they do not produce equilibrium. That is, these two forces do not balance each other. (In the previous section when we were weighing an object with a spring scale, the spring force *balanced* the gravitational force. Both forces were exerted on the same object, and did produce equilibrium.)

Newton's laws and reference frames. Newton's laws are valid only in some reference frames. For example, imagine a ball hanging from a string inside a railroad car. (See diagram on next page.) When the car is at rest, the ball hangs straight down. The ball is at rest as seen from inside the railroad car, and the net force on it is zero also, thus confirming Newton's 2nd law. When the car is moving with constant velocity, the ball also hangs straight down. As seen from the ground the ball is moving, but as seen from inside the car it is at rest. The net force on the ball is zero, so Newton's 2nd law is again confirmed in both reference frames. However, when the railroad car is accelerating, such as when the wheels are locked in place, the ball hangs at an angle. As seen from the ground, the ball is accelerating, as predicted by Newton's 2nd law. But observations made from inside the railroad car contradict Newton's 2nd law, because now there is clearly a net force on the ball, yet it appears to be at rest inside the car. (Thus, $\mathbf{F}_{\text{net}} \neq m\mathbf{a} = 0$.)



three different reference frames for observing a ball hanging from a string

A reference frame in which Newton's laws are valid is called an *inertial frame*. The earth is only approximately an inertial frame, because objects that appear to be at rest on the earth are actually accelerating very slightly. The sun is a better inertial frame.

Newton's laws and free-body diagrams. To understand the motion of objects, we usually use Newton's 2nd law written in component form, as shown here:

$$F_{\text{net},x} = ma_x \qquad\qquad F_{\text{net},y} = ma_y$$

where:

$F_{\text{net},x}$ = the component of the net force in the x-direction
= the sum of the x-components of all the individual forces

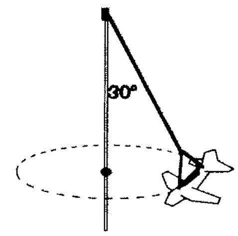
a_x = the component of the acceleration in the x-direction

$F_{\text{net},y}$ = the component of the net force in the y-direction
= the sum of the y-components of all the individual forces

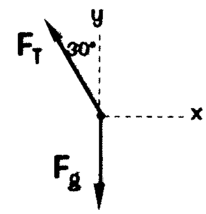
a_y = the component of the acceleration in the y-direction

If a coordinate system is clearly indicated and oriented on the free-body diagram, it is often straightforward to find the components of each of the forces that make up the net force. Let's go back to our example of the amusement-park ride:

An amusement park ride consists of an airplane on the end of a cable whirled in a horizontal circle. What is the net force on the airplane in the horizontal and vertical directions?



Answer. A free-body diagram is shown to the right. Only the tension force has a component in the horizontal (x) direction. It is ($-F_T$ sin30°). So, the x-component of the net force on the airplane is:



$$F_{\text{net},x} = -F_T \sin 30°$$

Both forces have components in the vertical (y) direction, so the y-component of the net force is:

$$F_{\text{net},y} = F_T \cos 30° + (-F_g) = F_T \cos 30° - F_g$$

In the following pages, we show how dynamics is used either to predict the behavior of objects or to learn about the physical world.

2.3 DYNAMICS

Briefly stated, *dynamics* is the study of the relationship between forces and motion. Using kinematics, we are able to recognize and distinguish situations in which objects are accelerating and those in which they are not. Dynamics enables us to relate the forces exerted on different objects in different situations to the masses of those objects and their motion.

An agenda for dynamics. If we want to be able to say how the motion of an object is affected by the things it interacts with in its surroundings, there are several steps we need to be able to take:

(1) *Identify forces / interactions.* We must be able to recognize and identify the different kinds of interactions that might occur between objects. In particular, because we express these interactions as forces, we need to know how to determine the direction in which each force is exerted.

(2) *Measure magnitudes of forces.* We must be able to determine how strongly an interaction affects an object in different situations. In other words, we need to measure forces. (We usually use springs to do this.)

(3) *Determine / apply / re-evaluate the force laws.* We must find out what each of the force laws depends upon (such as mass, speed, displacement, etc.) so that we can calculate them in different situations. (Of course, we do not make up these rules.